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2001 J. Phys. A: Math. Gen. 34 8851

(http://iopscience.iop.org/0305-4470/34/42/309)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **34** (2001) 8851–8857 PII: S0305-4470(01)25346-3

Remarks on the canonical quantization of noncommutative theories

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Received 25 May 2001, in final form 28 August 2001
Published 12 October 2001
Online at stacks.iop.org/JPhysA/34/8851

Abstract

Free noncommutative fields constitute a natural and interesting example of constrained theories with higher derivatives. The quantization methods involving constraints in the higher derivative formalism can be nicely applied to these systems. We study real and complex free noncommutative scalar fields where momenta have an infinite number of terms. We show that these expressions can be summed in a closed way and lead to a set of Dirac brackets which matches the usual corresponding brackets of the commutative case.

PACS numbers: 11.25−w, 02.20.−a, 02.40.Gh

1. Introduction

Recently, there has been a great deal of interest in noncommutative fields. This interest started when it was noted that noncommutative spaces naturally arise in perturbative string theory with a constant background magnetic field in the presence of *D*-branes. In this limit, the dynamics of the *D*-brane can be described by a noncommutative gauge theory [1]. Besides their origin in strings and branes, noncommutative field theories are a very interesting subject in their own right (for a general review of noncommutative field theory, see [2]). They have been extensively studied under several approaches [3, 4]. To obtain the noncommutative version of a field theory one essentially replaces the usual product of fields by the Moyal product [1, 2], which leads to an infinite number of spacetime derivatives over the fields. It can be directly verified that the Moyal product does not alter quadratic terms in the action, provided boundary terms are discarded. In this way, the noncommutativity does not affect the equations of motion for free fields. However, we know that momenta can be obtained as surface terms of a hypersurface orthogonal to the time direction [5]. This means that momenta are different in the versions with and without Moyal products. In fact, momenta in the version with Moyal products have an infinite number of terms.

Hence, noncommutative field theories provide us with an interesting and nonacademic example involving higher derivatives, where the quantization rules for such systems can be nicely applied. We emphasize that this is a very peculiar situation. Usually, nontrivial examples of systems with higher derivatives are just academic and plagued with ghosts and nonunitarity problems. In fact, one can say that these systems have never constituted a confident test for the nonconventional quantization procedure involving higher derivatives, mainly in the cases where there are constraints. Free noncommutative theories, even though they can be described without the Moyal product, are then an interesting theoretical laboratory for using the higher derivative formalism with constraints. This is precisely the purpose of our paper. We are going to study the quantization of the free noncommutative scalar theory without discarding the infinite higher derivative terms of the Lagrangian. We shall see that, regardless of the completely different expressions of the momenta, the canonical quantization can be consistently developed in terms of a constraint formalism.

Our paper is organized as follows. In section 2 we deal with free noncommutative real scalar fields and in section 3 we consider the complex case, where the momentum expressions are still more evolved. Section 4 contains our conclusions.

2. Real scalar fields

Let us consider the action

$$S = \frac{1}{2}\int_{t_0}^{t} \mathrm{d}t \int \mathrm{d}^3\vec{x}\; \partial_\mu\phi \star \partial^\mu\phi \tag{2.1}$$

where $\star$ is the notation for the Moyal product, whose definition for two general fields ϕ_1 and ϕ_2 reads

$$\phi_1(x)\star\phi_2(x) = \exp\Big(\frac{\mathrm{i}}{2}\,\theta^{\mu\nu}\partial^x_\mu\partial^y_\nu\Big)\phi_1(x)\phi_2(y)|_{x=y} \tag{2.2}$$

and $\theta^{\mu\nu}$ is a constant antisymmetric matrix.

We notice that if one discards the surface terms, the derivatives of the Moyal product will not contribute to the action (2.1). Concerning the equation of motion, this can be done without further considerations. However, for the momenta we observe that any two situations are completely different from each other. For systems with higher derivatives, velocities are unconventionally assumed to be independent coordinates. (For a detailed discussion on systems with higher derivatives, see [6] and for an application of this procedure, see [7].) In the particular case of action (2.1), the independent coordinates are ϕ and $\dot{\phi}$. It is accepted that there is a conjugate momentum for each of them. These can be obtained by fixing the variartion of fields and velocities at just one of the extreme times, say $\delta\phi(\vec{x},t_0) = 0 = \delta\dot{\phi}(\vec{x},t_0)$, and keeping the other extreme free [5]. The momenta conjugate to ϕ and $\dot{\phi}$ are the coefficients of $\delta\phi$ and $\delta\dot{\phi}$, respectively, taken at time t over a hypersurface orthogonal to t.

Considering the variation of the action with just the extreme in t_0 kept fixed, we have

$$\begin{aligned}\delta S &= \frac{1}{2}\int_{t_0}^{t}\mathrm{d}t\int \mathrm{d}^3\vec{x}\big(\partial_\mu\delta\phi\star\partial^\mu\phi + \partial_\mu\phi\star\partial^\mu\delta\phi\big)\\ &= \frac{1}{2}\int_{t_0}^{t}\mathrm{d}t\int\mathrm{d}^3\vec{x}\;\partial^\mu(\delta\phi\star\partial_\mu\phi + \partial_\mu\phi\star\delta\phi) = \frac{1}{2}\int\mathrm{d}^3\vec{x}(\delta\phi\star\dot{\phi} + \dot{\phi}\star\delta\phi)\end{aligned} \tag{2.3}$$

where the on-shell condition was used. The momenta shall be obtained from the development of (2.3). Using the definition of the Moyal product, we have

$$\delta S = \int \mathrm{d}^3\vec{x}\, \Big[\dot{\phi}\,\delta\phi + \frac{1}{2}\Big(\frac{\mathrm{i}}{2}\Big)^2 \theta^{\mu\nu}\theta^{\alpha\beta}\partial_\mu\partial_\alpha\dot{\phi}\,\partial_\nu\partial_\beta\,\delta\phi + \frac{1}{4!}\Big(\frac{\mathrm{i}}{2}\Big)^4 \theta^{\mu\nu}\theta^{\alpha\beta}\theta^{\rho\gamma}\theta^{\xi\eta}\partial_\mu\partial_\alpha\partial_\rho\partial_\xi\dot{\phi}\,\partial_\nu\partial_\beta\partial_\gamma\partial_\eta\,\delta\phi + \cdots\Big]. \tag{2.4}$$

We observe that the odd terms in $\theta^{\mu\nu}$ were cancelled in the expression above. This was because the symmetric terms that appear in the first step in (2.3) are actually necessary in the noncommutative case. In addition, due to the integration over $\mathrm{d}^3\vec{x}$, only terms in θ^{0i} will survive in the Moyal product[1]. For the quadratic term in $\theta^{\mu\nu}$, we obtain

$$\int \mathrm{d}^3\vec{x}\,\theta^{\mu\nu}\theta^{\alpha\beta}\partial_\mu\partial_\alpha\dot{\phi}\,\partial_\nu\partial_\beta\delta\phi = \int \mathrm{d}^3\vec{x}\,(\bar{\partial}^2\dddot{\phi}\,\delta\phi + 2\,\bar{\partial}^2\ddot{\phi}\,\delta\dot{\phi} + \bar{\partial}^2\dot{\phi}\,\delta\ddot{\phi}) \tag{2.5}$$

where we have used the short notation $\bar{\partial} = \theta^{0i}\partial_i$. Similarly, for the next term in the expression (2.4), we have

$$\int \mathrm{d}^3\vec{x}\;\theta^{\mu\nu}\theta^{\alpha\beta}\theta^{\rho\gamma}\theta^{\xi\eta}\partial_\mu\partial_\alpha\partial_\rho\partial_\xi\dot{\phi}\,\partial_\nu\partial_\beta\partial_\gamma\partial_\eta\delta\phi = \int \mathrm{d}^3\vec{x}\Big(\bar{\partial}^4\overset{(\mathrm{v})}{\phi}\,\delta\phi + 4\,\bar{\partial}^4\overset{(\mathrm{iv})}{\phi}\,\delta\dot{\phi} + 6\,\bar{\partial}^4\dddot{\phi}\,\delta\ddot{\phi} + 4\,\bar{\partial}^4\ddot{\phi}\,\delta\,\dddot{\phi} + \bar{\partial}^4\dot{\phi}\,\delta\,\overset{(\mathrm{iv})}{\phi}\Big) \tag{2.6}$$

where $\overset{(n)}{\phi}$ means n-time derivative over ϕ. We observe that the general rule to obtain other terms can be inferred. Introducing these results into the initial equation (2.4), grouping the coefficients of $\delta\phi$, $\delta\dot{\phi}$, $\delta\ddot{\phi}$, etc and writing each term in a more convenient way, we have

$$\begin{aligned}\delta S = \int \mathrm{d}^3\vec{x}\,\Big\{&\Big[\binom{0}{0}\dot{\phi} + \binom{2}{0}\frac{1}{2!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^2\dddot{\phi} + \binom{4}{0}\frac{1}{4!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^4\overset{(\mathrm{v})}{\phi} + \cdots\Big]\delta\phi\\ &+\Big[\binom{2}{1}\frac{1}{2!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^2\ddot{\phi} + \binom{4}{1}\frac{1}{4!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^4\overset{(\mathrm{iv})}{\phi} + \cdots\Big]\delta\dot{\phi}\\ &+\Big[\binom{2}{2}\frac{1}{2!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^2\dot{\phi} + \binom{4}{2}\frac{1}{4!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^4\dddot{\phi} + \cdots\Big]\delta\ddot{\phi}\\ &+\Big[\binom{4}{3}\frac{1}{4!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^4\ddot{\phi} + \binom{6}{3}\frac{1}{6!}\Big(\frac{\mathrm{i}}{2}\bar{\partial}\Big)^6\overset{(\mathrm{iv})}{\phi} + \cdots\Big]\delta\dddot{\phi} + \cdots\Big\}\end{aligned} \tag{2.7}$$

where

$$\binom{p}{n} = \frac{p!}{n!(p-n)!} \tag{2.8}$$

We can rewrite this relation in a compact form as

$$\delta S = \int \mathrm{d}^3\vec{x}\sum_{p,n=0}^{\infty}\left[\frac{\big(\frac{\mathrm{i}}{2}\bar{\partial}\big)^{2p}\overset{(2p-2n+1)}{\phi}}{(2n)!(2p-2n)!}\delta\overset{(2n)}{\phi} + \frac{\big(\frac{\mathrm{i}}{2}\bar{\partial}\big)^{2p+2}\overset{(2p-2n+2)}{\phi}}{(2n+1)!(2p-2n+1)!}\delta\overset{(2n+1)}{\phi}\right]. \tag{2.9}$$

One cannot still infer the momenta from the expression above because all $\delta\overset{(n)}{\phi}$ are not independent. In fact, by virtue of the equations of motion we have, for example, $\delta\ddot{\phi} = \nabla^2\delta\phi$, $\delta\dddot{\phi} = \nabla^2\delta\dot{\phi}$ and so on. Using these on-shell conditions [5] in expression (2.9) and rewriting

[1] It was pointed out by Gomis and Mehen [4], that θ^{0i} should be taken as zero in the vertex terms in order to avoid causality and unitarity problems. However, the role played by these terms in the free case is not the same.

it in terms of even and odd n, it is then finally possible to identify the independent canonical momenta π and $\pi^{(1)}$, conjugate respectively to ϕ and $\dot{\phi}$,

$$\pi = \sum_{p,n=0}^{\infty} \frac{\left(\frac{\mathrm{i}}{2}\bar{\partial}\sqrt{\nabla^2}\right)^{2p+2n}\dot{\phi}}{(2p)!(2n)!} \tag{2.10}$$

$$\pi^{(1)} = \sum_{p,n=0}^{\infty} \frac{\left(\frac{\mathrm{i}}{2}\bar{\partial}\sqrt{\nabla^2}\right)^{2p+2n+2}\phi}{(2p+1)!(2n+1)!}. \tag{2.11}$$

An interesting point is that a careful analysis of the expressions above permits us to see that they can be cast in a closed form. We just write down the final result

$$\pi = \tfrac{1}{2}\left[1 + \cosh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\right]\dot{\phi} \tag{2.12}$$

$$\pi^{(1)} = \tfrac{1}{2}\left[-1 + \cosh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\right]\phi. \tag{2.13}$$

The commutators cannot be directly obtained from the fundamental Poisson brackets

$$\{\phi(\vec{x},t), \pi(\vec{y},t)\} = \delta(\vec{x}-\vec{y}) \tag{2.14}$$

$$\left\{\dot{\phi}(\vec{x},t), \pi^{(1)}(\vec{y},t)\right\} = \delta(\vec{x}-\vec{y}) \tag{2.15}$$

because both relations (2.12) and (2.13) are constraints. The calculation of the Dirac brackets can be done in a direct way (see appendix A). The most relevant bracket for obtaining the propagator and the remaining quantization procedure is

$$\{\phi(\vec{x},t), \dot{\phi}(\vec{y},t)\}_D = \delta(\vec{x}-\vec{y}) \tag{2.16}$$

which means that the canonical quantization, even starting from the nontrivial momentum expressions (2.10) and (2.11), or (2.12) and (2.13), leads to the same result of the corresponding free commutative theory.

3. Complex scalar fields

We have seen in the previous analysis that there was a cancellation of odd terms in $\theta^{\mu\nu}$ in the δS on-shell variation given by (2.3). This was due to the symmetry of the real scalar fields in the action (2.1). In this section we are going to consider complex scalar fields where this symmetry does not exist and consequently the cancellation no longer occurs.

The noncommutative action for complex fields reads

$$S = \int \mathrm{d}^4x\, \partial_\mu\phi^* \star \partial^\mu\phi. \tag{3.1}$$

We could have also written here a symmetric quantity by adding a term with $\partial_\mu\phi \star \partial^\mu\phi^*$ into the Lagrangian of the action (3.1). We are going to see that this is nonetheless necessary because the action with $\partial_\mu\phi \star \partial^\mu\phi^*$, even though having different momenta expressions, leads to the same quantum result as that given by (3.1). What is important to notice is that the Lagrangian of the action (3.1) does not have any problem related to Hermiticity, i.e., $(\partial_\mu\phi^* \star \partial^\mu\phi)^* = \partial_\mu\phi^* \star \partial^\mu\phi$, and discarding boundary terms, the action (3.1) leads to the usual free case $S = \int \mathrm{d}^4x\, \partial_\mu\phi^*\partial^\mu\phi$.

Following the same steps as those in section 2, we get

$$\delta S = \int \mathrm{d}^3\vec{x}(\delta\phi^* \star \dot{\phi} + \dot{\phi}^* \star \delta\phi) \tag{3.2}$$

where a similar development permits us to obtain the momenta

$$\pi = \sum_{p,n=0}^{\infty} \frac{1}{p!(2n)!} \left(-\frac{\mathrm{i}}{2}\bar{\partial}\sqrt{\nabla^2}\right)^{p+2n} \dot{\phi}^* \tag{3.3}$$

$$\pi^{(1)} = \sum_{p,n=0}^{\infty} \frac{1}{p!(2n+1)!} \left(-\frac{\mathrm{i}}{2}\bar{\partial}\sqrt{\nabla^2}\right)^{p+2n+1} \phi^* \tag{3.4}$$

$$\pi^* = \sum_{p,n=0}^{\infty} \frac{1}{p!(2n)!} \left(\frac{\mathrm{i}}{2}\bar{\partial}\sqrt{\nabla^2}\right)^{p+2n} \phi \tag{3.5}$$

$$\pi^{(1)*} = \sum_{p,n=0}^{\infty} \frac{1}{p!(2n+1)!} \left(\frac{\mathrm{i}}{2}\bar{\partial}\right)^{p+2n+1} \phi \tag{3.6}$$

respectively, conjugate to ϕ, $\dot{\phi}$, ϕ^* and $\dot{\phi}^*$. Also here, these sums lead to closed expressions

$$\pi = \tfrac{1}{2}\Big[1 + \cosh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\Big]\dot{\phi}^* - \tfrac{1}{2}\sinh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\sqrt{\nabla^2}\,\phi^* \tag{3.7}$$

$$\pi^{(1)} = \tfrac{1}{2}\Big[-1 + \cosh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\Big]\phi^* - \tfrac{1}{2}\sinh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\frac{1}{\sqrt{\nabla^2}}\,\dot{\phi}^* \tag{3.8}$$

$$\pi^* = \tfrac{1}{2}\Big[1 + \cosh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\Big]\dot{\phi} + \tfrac{1}{2}\sinh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\sqrt{\nabla^2}\,\phi \tag{3.9}$$

$$\pi^{(1)} = \tfrac{1}{2}\Big[-1 + \cosh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\Big]\phi + \tfrac{1}{2}\sinh\Big(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\Big)\frac{1}{\sqrt{\nabla^2}}\,\dot{\phi} \tag{3.10}$$

where the sinh-operators come from the odd terms in $\theta^{\mu\nu}$. All the relations above are constraints. The calculation of the Dirac brackets is a kind of direct algebraic work (see appendix B). The important point is that the brackets

$$\begin{aligned} \{\phi(\vec{x},t), \dot{\phi}^*(\vec{y},t)\}_D &= \delta(\vec{x}-\vec{y}) \\ \{\phi^*(\vec{x},t), \dot{\phi}(\vec{y},t)\}_D &= \delta(\vec{x}-\vec{y}) \end{aligned} \tag{3.11}$$

are obtained, which means that the canonical quantization is correctly achieved.

To conclude this section, let us mention that we could have started from

$$\tilde{S} = \int \mathrm{d}^4x\, \partial_\mu\phi \star \partial^\mu\phi^* \tag{3.12}$$

instead of the action (3.1). Considering the on-shell variation of $\tilde{S}$ and keeping one of the extreme times fixed, we have

$$\delta\tilde{S} = \int \mathrm{d}^3\vec{x}\,(\delta\phi \star \dot{\phi}^* + \dot{\phi} \star \delta\phi^*) \tag{3.13}$$

which leads to expressions for the momenta similar to (3.7)–(3.10) with a change in the sign of the sinh-terms. Even though the expressions for the momenta are not equivalent in the two cases, we can trivially show that the constrained canonical procedure leads to the same Dirac brackets given by (3.11).

4. Conclusions

We have studied the free noncommutative scalar theory by using the constrained canonical formalism in the appropriate form for dealing with higher order derivative theories. This means that we have considered the momenta as defined as the coefficients of $\delta\phi$ and $\delta\dot{\phi}$ calculated on the hypersurface orthogonal to the time direction. We have shown that the evolved expressions coming from the momenta definitions can be summed in a closed way making it possible to be harmoniously applied in the Dirac constrained formalism. We have also considered the complex scalar fields, where the momentum expressions are still more evolved.

These examples naturally obtained from noncommutative theories make it possible to verify the consistency of the constrained canonical quantization procedure involving higher derivatives which is in some sense a controversial subject in the literature.

Acknowledgment

This work was supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico—CNPq (Brazilian Research agency) with the support of PRONEX 66.2002/1998-9.

Appendix A. Dirac brackets for the real scalar case

Let us denote constraints (2.12) and (2.13) in a simplified notation,

$$T^0 = \pi + K\,\dot\phi \tag{A.1}$$

$$T^1 = \pi^{(1)} + L\,\phi \tag{A.2}$$

where K and L are the operators

$$K = -\tfrac{1}{2}\left[1 + \cosh\left(\mathrm{i}\bar\partial\sqrt{\nabla^2}\right)\right] \tag{A.3}$$

$$L = \tfrac{1}{2}\left[1 - \cosh\left(\mathrm{i}\bar\partial\sqrt{\nabla^2}\right)\right] \tag{A.4}$$

Using the fundamental Poisson brackets given by (2.14) and (2.15), we have

$$\begin{aligned}\{T^0(\vec x,t),T^1(\vec y,t)\} &= -L_y\delta(\vec x-\vec y) + K_x\,\delta(\vec x-\vec y)\\ &= -\delta(\vec x-\vec y)\\ &= -\{T^1(\vec x,t),T^0(\vec y,t)\}\end{aligned} \tag{A.5}$$

where in the last step there was a providential cancellation of the even operators $\cosh\left(\mathrm{i}\bar\partial\sqrt{\nabla^2}\right)$ acting on $\delta(\vec x-\vec y)$. Since the remaining Poisson brackets of the constraints are zero, we have that the corresponding Poisson brackets matrix is given by

$$M = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\delta(\vec x-\vec y) \tag{A.6}$$

whose inverse is directly obtained, as is also the Dirac brackets (2.16).

Appendix B. Dirac brackets for the complex case

Let us denote the corresponding constraints by

$$\begin{aligned}T^0 &= \pi + K_1\,\dot\phi^* + K_2\,\phi^*\\ T^1 &= \pi^{(1)} + L_1\,\dot\phi^* + L_2\,\phi^*\\ T^2 &= \pi^* + K_1\,\dot\phi - K_2\,\phi\\ T^0 &= \pi^{(1)*} - L_1\,\dot\phi + L_2\,\phi\end{aligned} \tag{B.1}$$

where K_1, K_2, L_1 and L_2 are short notations for the operators

$$\begin{aligned}
K_1 &= -\tfrac{1}{2}\left[1+\cosh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\right] \\
K_2 &= \tfrac{1}{2}\sinh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\sqrt{\nabla^2} \\
L_1 &= \tfrac{1}{2}\sinh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\frac{1}{\sqrt{\nabla^2}} \\
L_2 &= \tfrac{1}{2}\left[1-\cosh\left(\mathrm{i}\bar{\partial}\sqrt{\nabla^2}\right)\right].
\end{aligned} \tag{B.2}$$

The Poisson brackets for these constraints are

$$\begin{aligned}
\{T^0(\vec{x},t), T^2(\vec{y},t)\} &= (K_{2y}+K_{2x})\,\delta(\vec{x}-\vec{y}) = 0 \\
\{T^0(\vec{x},t), T^3(\vec{y},t)\} &= (-L_{2y}+K_{1x})\,\delta(\vec{x}-\vec{y}) = -\delta(\vec{x}-\vec{y}) \\
\{T^1(\vec{x},t), T^2(\vec{y},t)\} &= (K_{1y}+L_{2x})\,\delta(\vec{x}-\vec{y}) = \delta(\vec{x}-\vec{y}) \\
\{T^1(\vec{x},t), T^3(\vec{y},t)\} &= (L_{1y}+L_{1x})\,\delta(\vec{x}-\vec{y}) = 0.
\end{aligned} \tag{B.3}$$

The remaining brackets are trivially zero. It is interesting to observe the harmonious cancellation among the different operators acting on the delta function. Now, we can easily construct the matrix of the Poisson brackets of the constraints and calculate the relevant Dirac brackets given by (3.11).

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